Name:_____ Date:_____

Pre Calculus 12 Section 2.4 Reciprocal Functions:

1. What does it mean to take the reciprocal of a number? Explain:

   To take the inverse of the #. → 2 is the same as $\frac{2}{1}$ and you take the inverse which is $\frac{1}{2}$.

2. If the coordinate (a,b) is on the function $y = f(x)$, what will this coordinate become on the function $y = \frac{1}{f(x)}$? $\left(a, \frac{1}{b}\right)$ where $b \neq 0$

3. When you take the reciprocal of a function, what happens to the equation?

   It becomes an asymtote

4. What is a vertical asymptote? How do you find the vertical asymptotes of a reciprocal function?

   L> A vertical line in which no point may touch.

   The x intercept(s) of a function become a vertical asymptotes

5. If you taking the reciprocal function, how many vertical asymptotes can you have? Explain:

   As many as the # of x intecepts

6. Suppose you are given a horizontal line $y = 5$, what would the reciprocal of this function become?

   $y = \frac{1}{5}$

7. Suppose you are given a linear function, $y = mx + b$, how do you find the domain and range of the reciprocal of this function? Explain:

   R: $y > 0$ $y < 0$ D: $x >$ x-int $x <$ x-int

8. Suppose you are given a quadratic function, $y = x^2 - k$, how do you find the domain and range of the reciprocal of this function? Explain:

   find the 2 x-int $< \begin{matrix} x_1 \\ x_2 \end{matrix}$ (labeling the x-int)

   R: $y > 0$ $y < 0$ D: $x < x_1$, $x_1 < x < x_2$, $x > x_2$)

9. Suppose you are given the following coordinates on $y=f(x)$, what will these coordinates become on $y=\dfrac{1}{f(x)}$?

$i(5,-1)$ ii $(3,-2)$ iii $(10,20)$ iv $\left(6,-\frac{2}{3}\right)$ v $\left(\frac{4}{7},\frac{9}{4}\right)$ vi $(0,-1)$ vi $(-1,0.004)$

$(5,-1)$ $(3,-\frac{1}{2})$ $(10,\frac{1}{20})$ $(6,-\frac{3}{2})$ $(\frac{4}{7},\frac{4}{9})$ $(0,-1)$ $(-1,250)$

$\frac{1}{0.004}=\frac{1000}{4}=250$

10. How do you find the invariant points of a reciprocal function? Explain:

Where $y=\pm 1$, those are the invariant points

11. Given each of the following reciprocal functions, Find the equation of the vertical asymptotes and the coordinates of ALL the invariant points?

a) $y=\dfrac{1}{5x-9}$

VA:

$y=5x-9$

$5x=9$

$x=\frac{9}{5}$

VA $\rightarrow$ $\boxed{x\neq\frac{9}{5}}$

Inv points

$\pm 1=5x-9$

$10=5x$ and $8=5x$

$\boxed{x=2}$ and $\boxed{x=\frac{8}{5}}$

b) $y=\dfrac{1}{\frac{2}{3}x-11}$

VA:

$0=\frac{2}{3}x-11$

$11=\frac{2}{3}x$

$\boxed{\frac{33}{2}\neq x}$

Inv points:

$\pm 1=\frac{2}{3}x-11$

$12=\frac{2}{3}x$ or $10=\frac{2}{3}x$

$\boxed{x=18}$ $\boxed{x=15}$

c) $y=\dfrac{1}{2x^2-9}$

VA:

$0=2x^2-9$

$9=2x^2$

$\frac{9}{2}=x^2$

$\pm\sqrt{\frac{9}{2}}=x \rightarrow \pm\frac{3}{\sqrt{2}} \rightarrow \boxed{\pm\frac{3\sqrt{2}}{2}}$

Inv points

$\pm 1=2x^2-9$

$2x^2=10$ $\quad x^2=5$ $\quad \boxed{x=\pm\sqrt{5}}$

$2x^2=8$ $\quad x^2=4$ $\quad \boxed{x=\pm 2}$

d) $y=\dfrac{1}{(x-3)^2-16}$

VA:

$0=(x-3)^2-16$

$16=(x-3)^2$

$\pm 4=x-3$

$\boxed{x=7}$ $\boxed{x=-1}$

INV points:

$\pm 1=(x-3)^2-16$

$15=(x-3)^2$ $\quad \boxed{x=3\pm\sqrt{15}}$

$17=(x-3)^2$ $\quad \boxed{x=3\pm\sqrt{17}}$

e) $y=\dfrac{1}{8x^2-10x-3}$

$0=8x^2-10x-3$

$x=\dfrac{10\pm\sqrt{100+96}}{16}$

$x=\dfrac{10\pm 14}{16}$

$x=\frac{24}{16}$ $\quad \boxed{x=\frac{3}{2}}$

$x=\frac{-4}{16}$ $\quad \boxed{x=-\frac{1}{4}}$

Inv points

$\pm 1=8x^2-10x-3$

$0=8x^2-10x-2$ $\quad 0=8x^2-10x-4$

$0=4x^2-5x-1$ $\quad 0=4x^2-5x-2$

$x=\frac{5\pm\sqrt{25+16}}{8}$ $\quad x=\frac{5\pm\sqrt{25+32}}{8}$

$\boxed{x=\frac{5\pm\sqrt{41}}{8}}$ $\quad \boxed{x=\frac{5\pm\sqrt{57}}{8}}$

f) $y=\dfrac{1}{2x^2-7x-15}$

$0=2x^2-7x-15$

$0=\left(\frac{2x-10}{2}\right)\left(\frac{2x+3}{1}\right)$ $\quad -30$, -10, 3, -7

$0=(x-5)(2x+3)$

$\boxed{x=5}$ $\quad \boxed{x=-\frac{3}{2}}$

Inv points

$\pm 1=2x^2-7x-15$

$0=2x^2-7x-14$ $\quad 2x^2-7x-16$

$x=\frac{7\pm\sqrt{49+112}}{4}$ $\quad x=\frac{7\pm\sqrt{49+128}}{4}$

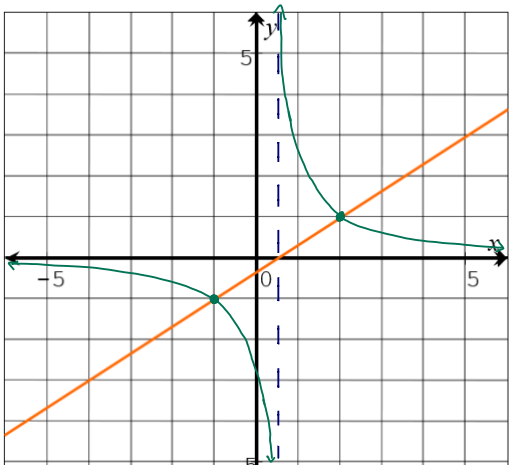
$\boxed{x=\frac{7\pm\sqrt{161}}{4}}$ $\quad \boxed{x=\frac{7\pm\sqrt{177}}{4}}$

12. Given the graph of $y = f(x)$, graph the reciprocal function $y = \dfrac{1}{f(x)}$ on the same graph. Label the vertical asymptotes and the invariant points on the graph:

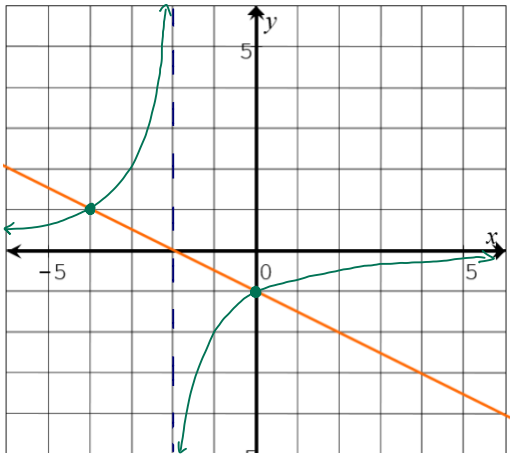
vertical asymptotes

Invariant points

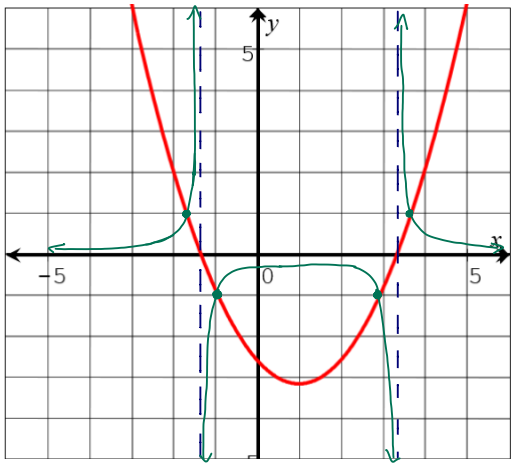
a)



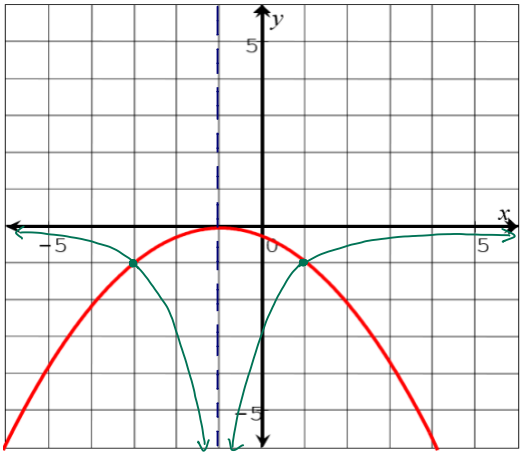
b)



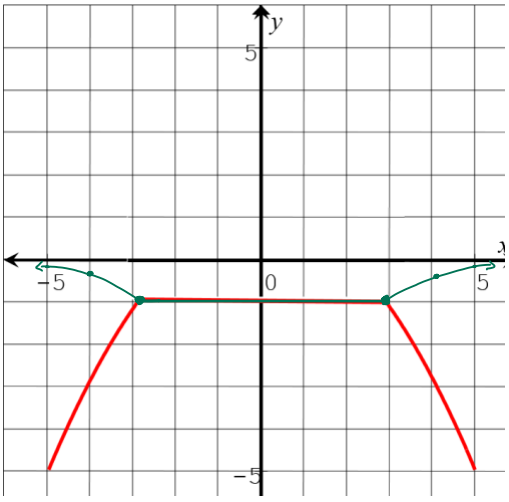
c)



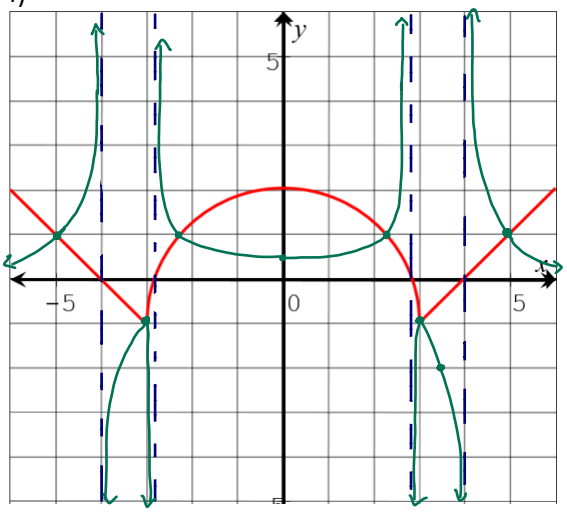
d)



e)



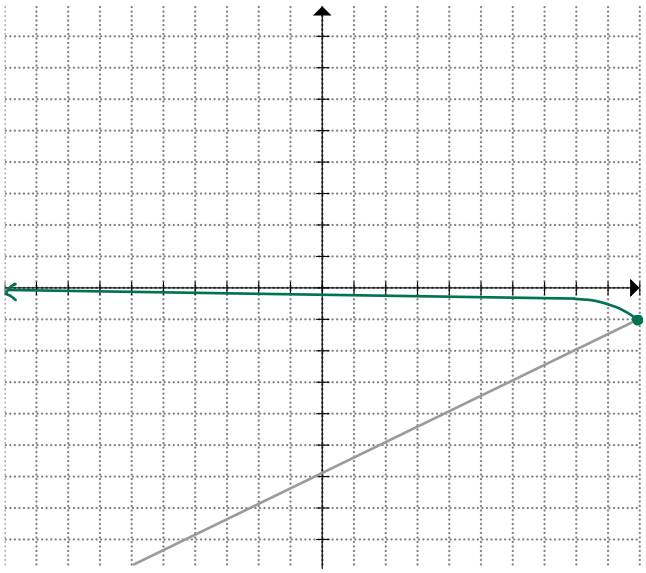
f)



13. Graph the each of the following reciprocal functions. Indicate the asymptotes, coordinates of the invariant points, domain and range:

$y = \dfrac{1}{\frac{1}{2}x - 6}$

Invariant Points:
(10 , -1) (14 , 1)

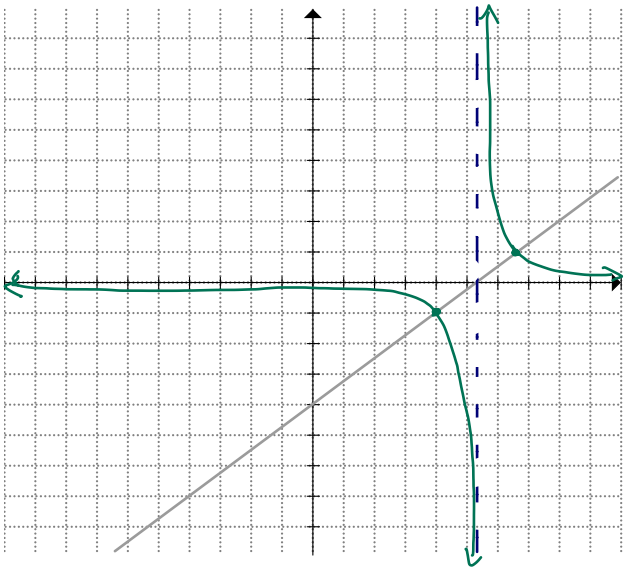


Domain and Range: D : $(x \mid x \in \mathbb{R},\ x \neq 12)$
R : $(y \mid y \in \mathbb{R},\ y \neq 0)$

Equations of Asymptotes: $y = 0$ ← horizontal
$x = 12$ ← vertical

$y = -\dfrac{1}{\frac{-3}{4}x + 4}$

Invariant Points
(4 , -1) $(\frac{20}{3}, 1)$

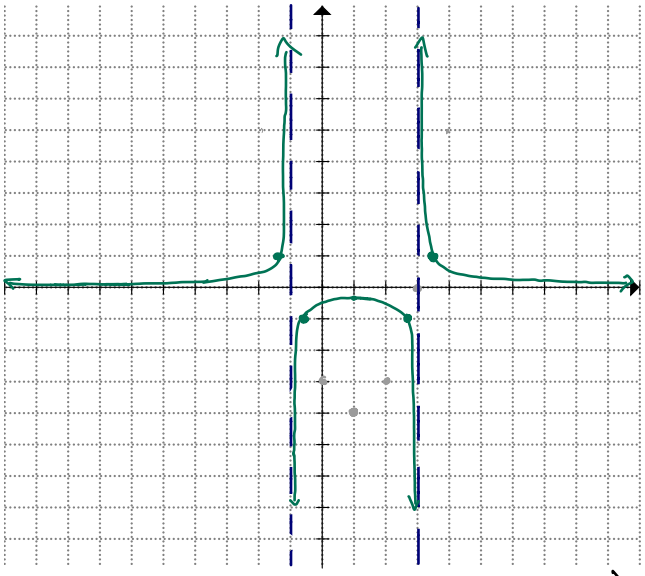


Domain and Range: D : $(x \mid x \in \mathbb{R},\ x \neq \frac{16}{3})$
R : $(y \mid y \in \mathbb{R},\ y \neq 0)$

Equations of Asymptotes $y = 0$ ← horizontal
$x = 12$ ← vertical

$y = \dfrac{1}{(x-1)^2 - 4}$

Invariant Points
$(1 \pm \sqrt{5}, 1)$ $(1 \pm \sqrt{3}, -1)$

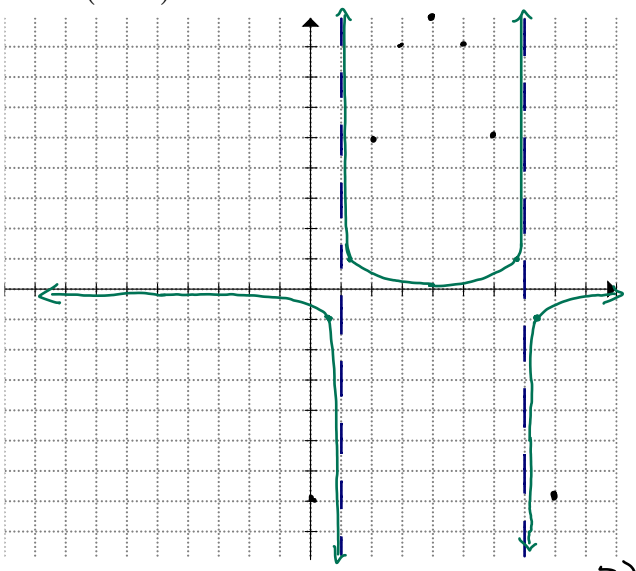


Domain and Range: D : $(x \mid x \in \mathbb{R}, x \neq -1,\ x \neq 3)$
R : $(y \mid y \in \mathbb{R},\ y > 0,\ y \leq -\frac{1}{4})$

Equations of Asymptotes: $y = 0$
$x_1 = -1$ $x_2 = 3$

$y = \dfrac{1}{-(x-4)^2 + 9}$

Invariant Points
$(4 \pm \sqrt{8}, 1)$, $(4 \pm \sqrt{10}, -1)$



Domain and Range: D : $(x \mid x \in \mathbb{R},\ x \neq 1, x \neq 7)$
R : $(y \mid y \in \mathbb{R},\ y < 0, y \geq \frac{1}{9})$

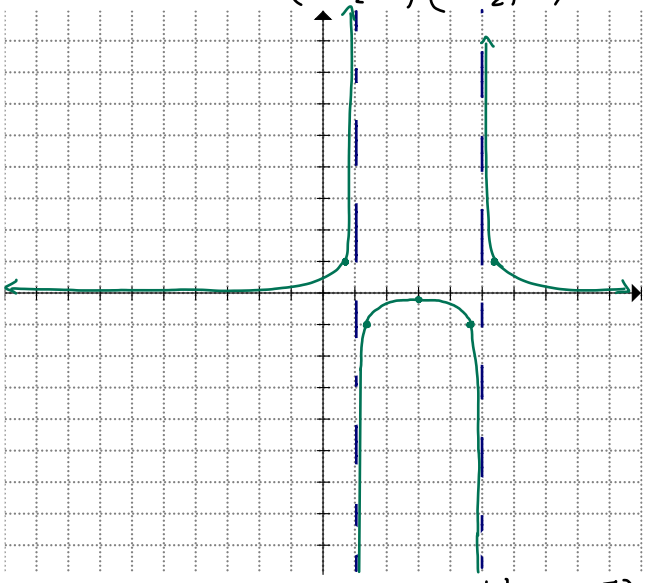
Equations of Asymptotes $y = 0$
$x = 1$ $x_2 = 7$

$-10-1=12x^2$ -11 $\frac{11}{2}=x^2$ $\frac{\sqrt{22}}{2}$

$1-10=-2x^2$ -9 $\frac{9}{2}=x^2$ $x=\pm\frac{3\sqrt{2}}{2}$

$y=\frac{1}{2(x-3)^2-8}$

Invariant Points: $(3\pm\frac{3\sqrt{2}}{2}, 1)$ $(3\pm\frac{\sqrt{14}}{2}, -1)$



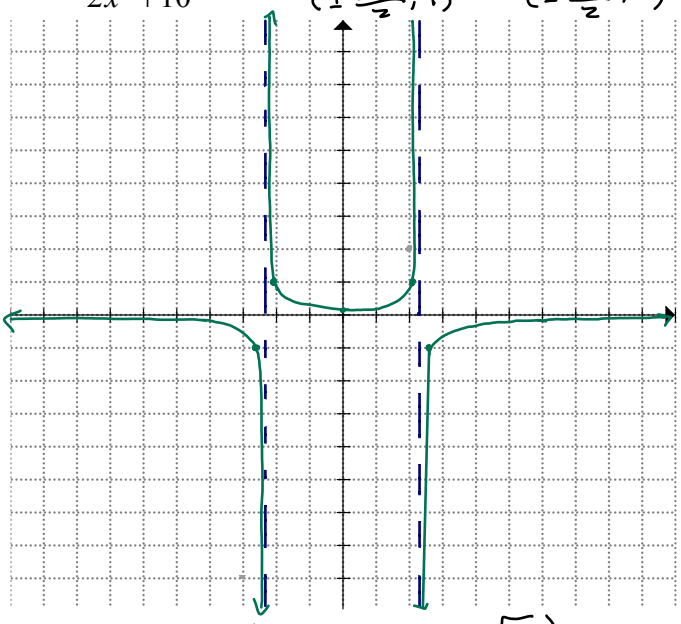
Domain and Range: D: $(x \mid x\in\mathbb{R}, x\neq 1, x\neq 5)$

R: $(y \mid y\in\mathbb{R}, y>0, y\leq\frac{1}{8})$

Equations of Asymptotes: $x=1$ $x=5$ $y=0$

$y=\frac{1}{-2x^2+10}$

Invariant Points: $(\pm\frac{3\sqrt{2}}{2}, 1)$ $(\pm\frac{\sqrt{22}}{2}, -1)$



Domain and Range: D: $(x \mid x\in\mathbb{R}, x\neq\pm\sqrt{5})$

R: $(y \mid y\in\mathbb{R}, y<0, y\geq\frac{1}{10})$

Equations of Asymptotes: $x=\pm\sqrt{5}$ $y=0$

14. Match each reciprocal function with the correct domain and range on the right:

Functions:	ANSWER			CHOICES	
	Domain	Range:		DOMAINS	RANGES:
a) $y=\frac{1}{3x-5}$	NONE domain: $x\neq\frac{5}{3}$	I)		i) $x\in\mathbb{R}$	I) $y\in\mathbb{R}; y\neq 0$
				ii) $x\in\mathbb{R}; x\neq 2$	II) $y\in\mathbb{R}; y\neq\frac{5}{3}$
b) $y=\frac{1}{2x-4}$	ii)	I)		iii) $x\in\mathbb{R}; x\neq 1,5$	III) $y\in\mathbb{R}$
				iv) $x\in\mathbb{R}; x\neq 5,-1$	
c) $y=\frac{1}{(x-2)^2-9}$	iv)	IV)		v) $x\in\mathbb{R}; x\neq\frac{3}{5}$	IV) $y>0, y\leq-\frac{1}{9}$
				vi) $x\in\mathbb{R}; x\neq -1,-7$	v) $y>0, y<-\frac{1}{9}$
d) $y=\frac{1}{(x-3)^2-4}$	iii) ix)	vi)		Vii) $x\in\mathbb{R}; x\neq\frac{1}{2}$	vi) $y>0, y\leq-\frac{1}{4}$
				viii) $x\in\mathbb{R}; x\neq 1,7$	vii) $y<0, y\geq\frac{1}{18}$
e) $y=\frac{1}{-2(x-4)^2+18}$	viii)	viii)		ix) $x\in\mathbb{R}; x\neq 1,5$	viii) $y>0, y\leq\frac{1}{9}$